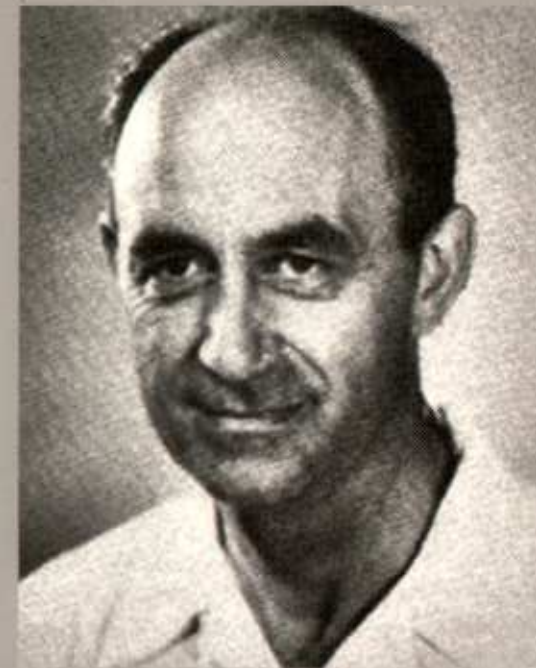
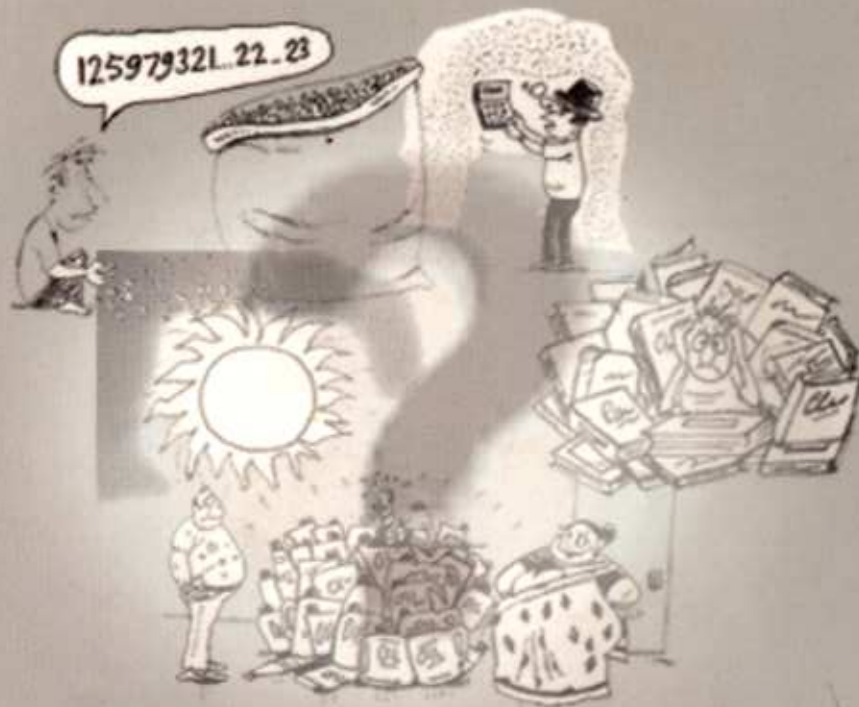


Fermi Problems or The Art of Estimation

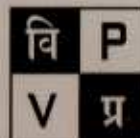


Vinay B. Kamble



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Or
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V. B . Kamble



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(Regd. Office : Technology Bhawan, New Delhi 110 016)

Ph : 0120-2404430-31 Fax : 0120-2404437

E-mail : info@vigyanprasas.gov.in

Internet : http://www.vigyanprasas.gov.in

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Fermi Problems Or The Art of Estimation

Author :- V. B. Kamble

Illustrations : *Irfan*

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Hans Christian Von Baeyer wrote an article entitled “How Fermi would have fixed it” which appeared in the October 1988 issue of *The Sciences*. Though familiar with Fermi problems for quite some time, it was only after reading the aforesaid article that made me think about the relevance and importance of them in our educational system. We often talk about integrated science and try to show how different disciplines of science come into play in an integrated manner in a scientific phenomenon. Often similar situations arise in social and economic problems.

Often we need some rough estimates, projections, impact of a new scheme etc., and we may not have sufficient data to go about. The Fermi approach plays a crucial role in such circumstances. With common sense, tit-bits of information and figures that everybody carries in his/her head, one can come to an estimate close to the answer to a problem. Provided one breaks up the seemingly difficult problems into small sub-problems which then become tractable. Of course, one needs to be familiar with some basic figures and facts.

Fermi problems widen one’s horizon and develop a sense of appreciation. What is more important, it helps understand the inter-relationship of different disciplines in a given problem, phenomenon or situation. It is therefore a training and experience that enriches our lives.

An attempt has been made here to illustrate the Fermi approach through some problems in physics and our socio-economic environment. There are some problems only for fun.

I have heavily drawn on the article by Baeyer mentioned earlier. Fermi problems have become a regular feature in competitive tests in Western countries. It is high time, our children, why, even you scientists embarking upon a scientific career became

familiar with Fermi problems and Fermi approach to tackle apparently difficult situations.

I am thankful to Dr. C. J. Sanchorawala and Prof. A.R. Rao, my former colleague at Vikram A. Sarabhai Community Science Centre, Ahmedabad, for discussing many of the problems given here. I am especially indebted to late Prof. P.R. Pisharoty, Professor Emeritus, Physical Research Laboratory, Ahmedabad and Prof. P.P. Kale, Former Director, Space Applications Centre, ISRO, (Ahmedabad) for many instructive discussions and critical suggestions for improvement of the manuscript.

This book was originally published in 1990 by Gujarat Science Academy, Ahmedabad. Hindi version of the book was published by Vigyan Prasar in 2004. A few illustrations have been added in the present edition.

Any suggestions to improve the booklet are welcome.

V.B. Kamble
Director
Vigyan Prasar

Fermi Problems Or The Art of Estimation

Fermi – Theoretician & Experimentalist par excellence

At twenty nine minutes past five, on a Monday morning in July of 1945, the world's first atom bomb exploded in the desert sixty miles northwest of Alamogordo, New Mexico. Forty seconds later, the blast's shock wave reached the base camp, where scientists stood in stunned contemplation of historic spectacle. The first person to stir was the Italian-American physicist Enrico Fermi, who was on hand to witness the culmination of a project he had begun.

Before the bomb detonated, Fermi had torn a sheet of notebook paper into small bits. Then as he felt the first quiver of the shock wave spreading outward through the still air, he released the shreds above his head. They fluttered down and away from the mushroom cloud growing on the horizon, landing about two and a half yards behind him. After a brief mental calculation, Fermi announced that the bomb's energy had been equivalent to that produced by ten thousand tonnes of TNT. The sophisticated instruments that measured shock wave's velocity and pressure and weeks of analysis confirmed Fermi's instant estimate ! This was the genius of Enrico Fermi, the winner of Nobel Prize in 1938 for his work in elementary particle physics. Four years later he had produced the first sustained nuclear reaction in Chicago, ushering in the era of atomic weapons and commercial nuclear power. No other physicist of his generation, and no one since, has been at once a master experimentalist and a leading theoretician.

Fermi had a preference for the most direct, rather than the most intellectually elegant route to an answer. To illustrate this approach, imagine that the volume of an irregular object, say Earth is to be determined which is slightly pear shaped. A scientist may try to look up a formula, or search through the literature or even try to derive one. Alternatively, he could do what Fermi would have done, i.e. compute the volume numerically. Instead of relying on a formula, he

one person could divide the Earth into a large number of tiny cubes, the volume of which can be easily computed and added together. True, this method gives only approximate solution, but it is a method which is sure to produce the result. It can give a fairly correct estimate rather than an exact solution.

The Fermi technique essentially consists of dividing a difficult problem into small manageable ones. This method is certainly a rough and ready *modus operandi*. He passed it on to his students and developed a type of questions which are known by his name.

II. Fermi Problems

Let us now discuss what a Fermi problem is. Upon first hearing it, one doesn't have even the faintest notion what the answer might be. It might even appear to be lacking in basic information. Yet, when the problem is broken down into sub-problems, each one answerable without the help of experts or reference books, an estimate can be made, either mentally or on the back of an envelope, that comes remarkably close to the exact solution.

To illustrate the method, let us take a famous problem Fermi posed to his classes at the University of Chicago. The problem is, how many piano tuners are there in Chicago? At the outset, the problem does appear a bit too whimsical and quite improbable that anyone knows the answer. Of course, there is no standard solution, but assumptions can be made that lead to a solution.

Suppose the population of metropolitan Chicago is three million. An average family consists of four people. Piano being quite a popular musical instrument, one may further assume that one third of all families own pianos. Thus there are about 2,50,000 pianos in the city. If every piano is tuned every ten years, there are 25,000 tunings a year. If each tuner can service four pianos a day, two hundred and fifty days a year, for a total of 1000 tunings a year, there must be about 25 piano tuners in the city! The answer of course is not exact. It could be as low as 10 or as high as 50, but, as the yellow pages of the telephone directory showed, this number certainly fell in the correct range.

Thus, even if the order of magnitude of the answer is unknown, one can proceed on the basis of different assumptions and still arrive at estimates that fall within the range of the answer.

III. Why do we land up with estimates that fall within the range of answer?

The reason why we land up with estimates that fall within the range of answers is that, in any string of calculations, errors tend to cancel out one another. For example, in the piano problem, if someone assumed that every sixth rather than third family owns a piano, he is just as likely to assume that pianos are tuned every five, not ten years. It is as improbable that all of one's errors will be underestimates or overestimates as it is that all the throws in a series of coin tosses will be heads or tails. **The law of probabilities dictates that deviations from the correct assumptions will tend to compensate for one another**, so the final result tends to converge to the right answer.

IV. How Fermi problems differ from usual brain teasers?

The problem of determining number of piano tuners in the city of Chicago and other Fermi problems might seem to resemble brain teasers that appear in popular magazines. A typical example of a brain teaser is the following.

Given three containers that hold eight, five and three liters of milk respectively. How do you then divide 8 liters of milk in two equal parts using only the three containers?

A brain teaser such as the one described above can be solved through logical deductions with the data given in the statement of problem itself. Hence a brainteaser like this contains all the necessary information.

Fermi problems, on the other hand, differ markedly from the usual brain teasers in that, the answer to a Fermi problem cannot be verified by logical deductions alone and is always approximate. To know the exact number of piano tuners in Chicago, you may need to conduct a head count of all piano tuners in the city! All piano tuners may not even be listed in the yellow pages of the telephone directory. Further all piano tuners may not even own a telephone.

What is also important, solving a Fermi problem requires a knowledge of facts NOT mentioned in the statement of the problem.

It should now be pretty obvious that Fermi problems are more tied to the physical world unlike mathematical puzzles. By the same token, Fermi problems are reminiscent of the ordinary dilemma.

that non-physicists encounter everyday of their lives.

V. Fermi Problems: A habit that enriches our lives

Fermi problems that physicists face deal more with atoms, molecules and space than with pianos or number of television sets in a city. It is necessary to remember a few basic magnitudes, such as the approximate radius of a typical atom or the number of molecules in a litre of water or the radius of the Earth. For example, consider the distance a scooter must travel before a layer of rubber about the thickness of molecule is worn off the tread of its tyre. It turns out that a thickness roughly equal to a molecular diameter is removed in each revolution, i.e., one layer of molecules is lost every time the tyre turns once (see, Fermi problem No. 20).

There even could be problems which are social and economic in character concerning energy policy, water supply to a city, deforestation etc. Of course, solutions to such Fermi problems depend on more facts than average people, sometimes, even average scientists have on their fingertips. For those who do have them in mind, the calculation may take only a few minutes.

As remarked earlier, answers to Fermi problems tell us the range within which the actual results are likely to fall. Hence it is necessary to follow a golden principle: NEVER START A LENGTHY CALCULATION UNTIL YOU KNOW THE RANGE OF VALUES WITHIN WHICH THE ANSWER IS LIKELY TO FALL AND EQUALLY IMPORTANT, THE RANGE WITHIN WHICH THE ANSWER IS **UNLIKELY TO FALL**.

Fermi problems help scientists communicate with each other. Those accustomed to tackling Fermi problems approach the experiment or the subject as if their own, demonstrating their understanding by rough calculations. Everyone tries to arrive at the correct answer with the least effort. Fermi problems thus instill the spirit of independence in a person.

VI. A collection of Fermi Problems

We have seen in the preceding paragraphs that questions about atom bombs, piano tuners, automobile tyre etc., have little in common. But the manner in which they are solved is the same in every case and can be applied to questions outside the realm of physics. We find two basic types of responses. The fainthearted turn to authority – to reference books, bosses, expert consultants, physicians, ministers etc., while the independent minds use common

sense and the limited factual knowledge that everyone carries, make reasonable assumptions and derive their own, approximate solutions. Most of the common problems encountered in life can often be sorted out with nothing more than logic, common sense and patience.

Fermi problems and efforts to solve them impart a basic training to us to solve seemingly difficult problems and arrive at reasonable estimates. Solving them bolsters self confidence besides giving immense pleasure and pride that accompanies creativity. Do not look up an answer. Do not let anyone else find the answer for you. It will make you feel impoverished. The habit of treating a problem as a Fermi problem enriches one's life.

The few problems given in this booklet are only an assortment, taken from science, our social and economic environment and some everyday problems, meant for fun. No attempt was made to verify the answers, but, a few knowledgeable friends do say that the answers fall in the correct range.

The problems fall in three general categories viz., 1) Fermi problems in Physics; 2) Fermi problems in our social and economic environment; and 3) Fermi problems purely for fun! But all of them are meant for fun! The range of problems include the energy released per gram of charge in a fire cracker to number of trees that need to be cut off everyday to print newspapers in our country and to the number of bald-headed people in the world!

Hope this booklet sets a chain reaction cultivating a habit that enriches our lives and help approach personal dilemmas as Fermi problems. Do not try to look up hints and solutions before giving an honest try, lest you feel deprived of the sense of pleasure and pride that accompanies creativity!

FERMI PROBLEMS**Problem 1**

Suppose you do not remember the radius of the Earth. How will you determine the Earth's circumference without looking it up?

Hint :

You may use your knowledge of elementary geography and Earth's rotation about its own axis. Consider two places separated from each other by a large distance and located almost on the same latitude. If you also know the longitudes of both the places, how does this knowledge help estimate the circumference of the Earth?

Solution :

Consider two places almost on the same latitude such as Ahmedabad and Imphal separated from each other by about 2,500km. It is known that the Sun rises about 1 hour 30 minutes later in Ahmedabad than in Imphal.

1 hour 30 minutes correspond to $1/16^{\text{th}}$ of day. A day is the time the Earth takes to complete one revolution. So its circumference must be sixteen times two thousand and five hundred or 38,400 km. This is an answer that differs from the true value by less than four per cent.

Another way to arrive at the solution of this problem is given on page 28.

Problem 2

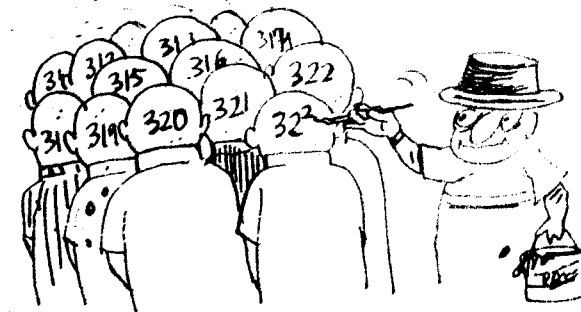
How many bald-headed people are there in the world?

Hint:

Find the population that is prone to develop bald. From this population, try to find out the age group that usually develops bald.

Solution:

The world population is about 6 billion. Assuming that women do not develop bald (!) and considering their number to be half, we are left with a population of 3 billion (males) that is prone to develop, bald. It is usual experience that people with bald are those more than 40 years of age. In all fairness, we may divide the population of 3 billion males into 3 age groups of 1 billion each. That is to say the age group 0-20, 20-40 and 40-60 years (and above) have 1 billion males each. The last group is the one containing bald headed people. An estimate of 10% bald headed people in this age group may not be unreasonable. Hence this leaves us with 100 million bald headed people in the world population of 6 billion people.



Problem 3

How many kgs of edible oil is consumed per year in an Indian city with population three million?

Hint :

Estimate how much edible oil is consumed in your family every year and then calculate for the entire population.

Solution :

A family of four on the average consumes about 5 kgs of edible oil per month, or about 15 kgs per head per year. True, this is a somewhat conservative estimate. Considering the population of the city (such as Ahmedabad) to be 3 million, the edible oil consumption per year would be 45 million kgs! Those of you familiar with packing of edible oil of 15 kgs tins, this means 3 million tins of 15 kgs edible oil are consumed every year by residents of an Indian city with population of three million!

**Problem 4**

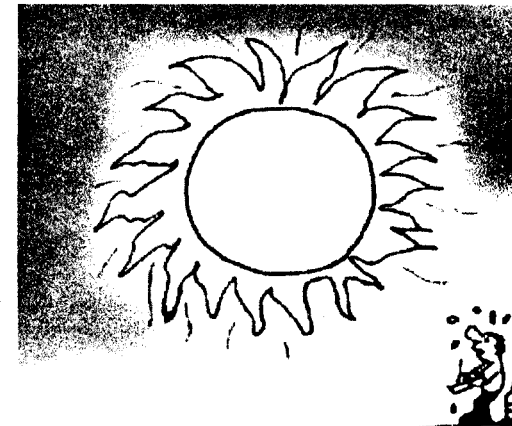
How could one determine the total power output of the Sun?

Hint:

You may need to know the solar flux (power incident per square meter) on the Earth—above the atmosphere to be precise. What area will you consider to calculate the total power output of the Sun?

Solution :

The solar flux, i.e. the incident solar power per square meter above the atmosphere is 1.36 kw/m^2 . The distance between the Sun and the Earth is 150 million km, or $15 \times 10^{10} \text{ m}$. To calculate the total power output of the Sun, we need to calculate the surface area of the sphere with radius $15 \times 10^{10} \text{ m}$, which is given by $4\pi r^2$, or $4 \times 3.14 \times (15 \times 10^{10})^2 \text{ m}^2$. This figure approximately would be $4 \times 3 \times 2.2 \times 10^{22}$, or $27 \times 10^{22} \text{ m}^2$. Multiplying by solar flux at Earth, we have $1.36 \times 27 \times 10^{22} \text{ kw}$, or about $3.5 \times 10^{23} \text{ kw}$, or $3.5 \times 10^{26} \text{ W}$ as the total power output of the Sun. A more accurate figure is $4 \times 10^{26} \text{ W}$.



Problem 5

How many molecules are there in the atmosphere? — One way.

Hint :

How will you estimate the volume of the atmosphere? You may now use a famous gas law to determine the number of molecules in the atmosphere.

Solution:

Consider the height of the atmosphere to be 20km, the density being uniform. The volume of the atmosphere can be calculated as follows.

The surface area of the Earth is $4\pi r_E^2$, where $r_E = 6 \times 10^6 \text{ m}$ is the radius of the Earth. This gives $4 \times 3 \times (6 \times 10^6)^2$, or about $3.6 \times 10^{14} \text{ m}^2$. The volume of atmosphere would therefore be $3.6 \times 10^{14} \times 20 \times 10^3 \text{ m}^3$ i.e. $7 \times 10^{18} \text{ m}^3$.

One mole has a volume of 22.4 litres at NTP or $22.4 \times 10^{-3} \text{ m}^3$, and the number of molecules in one mole is equal to Avogadro number or 6×10^{23} . Hence there are $6 \times 10^{23} / 22.4 \times 10^{-3}$ or about 25×10^{24} molecules/ m^3 . Thus the number of molecules in the atmosphere would be $25 \times 10^{24} \times 7 \times 10^{18}$ or about 2×10^{44} molecules.

Alternate solution to this problem is given on page 29.

**Problem 6**

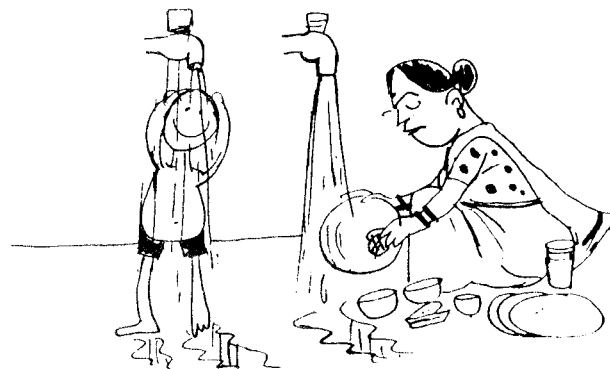
How much water do the residents of a metropolis with population three million use everyday?

Hint:

Try to guess how many buckets of water you may need per day for different jobs such as washing, bathing, cleaning etc., per day and then find the estimate for a population of three million.

Solution :

For a conservative estimate, let us consider you need two buckets of water (capacity 20 liters) for bath, two buckets for washing clothes, one bucket for cleaning utensils and yet another bucket for odd jobs, that is you need six buckets or 120 liters of water per day. Population of the city being three million, quantity of water used per day would be about 360 million liters.



Problem 7

How much water do people of a city of three million waste everyday?

Hint:

Estimate number of families, number of water connections and quantity of water supplied to each connection to find the water supplied to the entire population of the city with population three million.

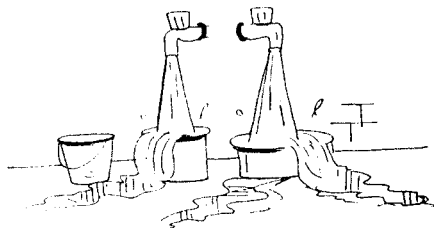
Solution :

Considering population to be three million, and a family consisting of four members, there are about 8,00,000 families in the city. Let us consider only half the number of families have a water connection, i.e. there are 4,00,000 water connections in the city.

It is possible to fill up about 30-40 buckets of water (capacity 20 litres) every hour (i.e. 2 minutes per bucket). Let us say, it is 30 buckets per hour. If water is supplied four hours a day, water supplied to the city per day would be $4 \times 10^5 \times 4 \times 30 \times 20$ or 96×10^7 . i.e. 1000 million litres per day approximately.

In Problem 6, we estimated that water used by the entire population of 3 million per day is 360 million litres. Hence water wasted per day would be 640 million litres, i.e. twice as much it is used!

Of course, our estimate is based on the assumption that the taps are NEVER closed (which is no doubt a great insult to the citizens!) Even if one assumes that the taps are closed, say for one hour, quantity of water supplied would be about 750 million litres and thus water wasted would be 390 million litres, i.e. almost as much as it is used!



Problem 8

How many wheat grains are there in a bag of 1 quintal?

Hint :

Estimate how many wheat grains could be there in 1 gm, then calculate for one quintal.

Solution:

The size of wheat grains can vary from variety to variety. Let us consider there are 10 grains in one gram of wheat. This gives 10×1000 or 10^4 grains in one kg of wheat. A quintal of course is 100kg. Hence there are 100×10^4 or one million grains in a bag of one quintal of wheat.



Problem 9

How many words are there in Encyclopaedia Britannica?

Hint:

Do you know the number of volumes of the Encyclopaedia Britannica and approximate size of one volume?

Solution:

Suppose there are about 1000 words in each page and that each volume has 1000 pages. This gives 10^6 as number of words per volume. Therefore in 20 volumes that comprise the Encyclopedia Britannica, the number of words would be 20×10^6 or 20 million!



Problem 10

How much gravitational force does a geosynchronous satellite weighing 1000 kg feel in orbit ?

Hint:

What would be the acceleration due to gravity at a height where geosynchronous satellite is situated?

Solution:

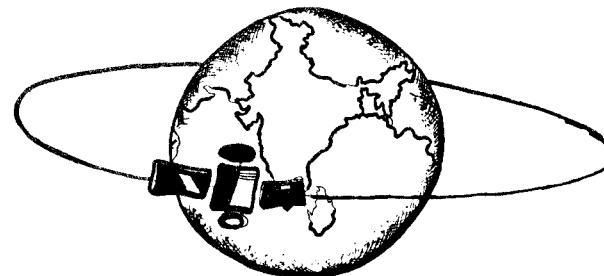
From the law of universal gravitation, we know that

$F = ma = mMG/r^2$. At the surface of the Earth, where $r=r_E$, the acceleration due to gravity would be

$$a = MG/r_E^2 = g = 10 \text{ m/s}^2$$

At an altitude of 36,000 kms above the surface of the Earth, $r=7r_E$ (since $r_E = 6,000$ km approximately). Hence $a=g/49$ or about 1/50 th of the acceleration due to gravity on surface of the Earth.

Hence, the gravitational force felt by a satellite of 1000 kg in a geosynchronous orbit would be $1000 \times 10/50 = 200\text{N}$ or just equal to 20 kg wt.



Problem 11

How many seconds make one year?

Hint :

Do you really need any?

Solution :

We have the following data.

Number of days in a year	= 365
Number of hours in day	= 24
Number of minutes in an hour	= 60
Number of seconds in one minute	= 60
Number of seconds in a day	= $24 \times 60 \times 60$
	= 24×3600
	$\approx 25 \times 4000$
	= 10^5

We might approximate the number of days in a year to 300.

$$\begin{aligned} \text{Hence number of seconds in one year} &= 10^5 \times 3 \times 10^2 \\ &= 3 \times 10^7 \text{ s.} \end{aligned}$$

**Problem 12**

How long will our Sun keep shining?

Hint:

What is the source of Sun's energy ? Calculate the energy produced using Einstein's mass-energy relationship.

Solution:

Unless we have some basic data, it is not possible to solve this problem in a straight forward manner.

We know that the source of Sun's energy is the conversion of hydrogen to helium. The sum of the masses of ingredients (four protons and two electrons) is about 1% greater than the mass product. The other information required is

Mass of the Sun	=	$2 \times 10^{30} \text{ kg}$
Energy output (See Fermi Problem No. 4)	=	$4 \times 10^{26} \text{ Watts}$
Speed of light 'C'	=	$3 \times 10^8 \text{ m/s}$

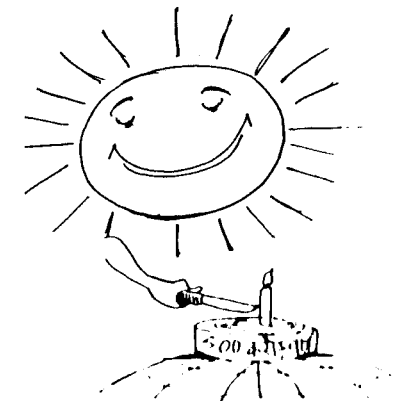
It may also be borne in mind that only 10% of the hydrogen is expected to be used up before hydrogen burning stops. We can now calculate the total energy output using Einstein's mass-energy relationship $E=mc^2$ during the life time of the Sun as follows.

$$E = 0.01 \times 0.1 \times 2 \times 10^{30} \times (3 \times 10^8)^2 = 1.8 \times 10^{44} \text{ joules}$$

At the rate of 4×10^{26} watts (joules/second), it will last about $1.8 \times 10^{44} / 4 \times 10^{26} = 4.5 \times 10^{17}$ s.

Now there are 3×10^7 seconds in one year (see Fermi problem No. 11). Hence the Sun would shine at the present rate for about 1.5×10^{10} or 15 billion years!!

It has already been shining for about 5 billion years!



Problem 13

Just before a fire-cracker was burst, a can weighing 0.250 kg was placed over it. It flew about 10 m high. How can you use this information to calculate the energy released per gm of carbon which is the main component of charge? This was an experiment actually carried out!

Hint:

Charge in a fire-cracker generally consists of carbon, sulphur and some nitrate compounds for oxidation. In a small sized cracker, it is about 1 gm or less.

Consider the work done on the can. What part of energy released do you think is used up in lifting the can?

Solution:

Work done on the can would be,

$$\begin{aligned} mgh &= 0.25 \times 10 \times 10 \\ &= 25 \text{ joules.} \end{aligned}$$

Considering the can plus cracker system as a heat engine, its efficiency cannot be greater than 1 or 2%. You may realize that this is reasonable estimate since the efficiency of a steam engine is also rarely more than 10-15%. Thus we may take the total energy released to be $25 \times 100 = 2500$ joules. 1 gm of charge may contain only about 0.2 or 0.3 gm of carbon, responsible for most energy release. Let us take it as 0.3g. Hence energy released per gram of carbon will be $2500/0.3 = 8.33$ KJ or about 10 KJ.

This information could also be used to calculate the energy released per gm mole of carbon. You may infer that it would be equal to $10 \times 12 = 120$ KJ. The actual figure is 393 KJ per gm mole. Our answer, however, off by a factor 3, gives a fairly correct estimate despite several experimental inaccuracies.

**Problem 14**

What is the speed with which an electron moves in an atom?

Hint:

What is the force acting on the electron? You may consider the case of hydrogen atom. Recall some universal constants.

Solution:

Considering the case of hydrogen atom, we have,
 $m_e V^2/r = (1/4\pi\epsilon_0) \times e^2/r^2$ the symbols denoting usual meanings. Recall the constants appearing here, or else look up a table of universal constants.

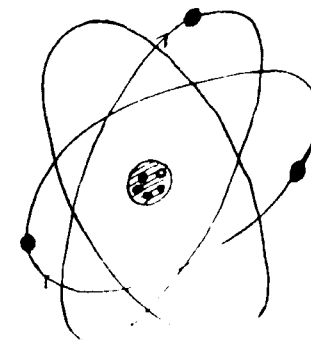
$$\begin{aligned} 4\pi &\approx 10 \\ m_e &= 9.11 \times 10^{-31} \text{ kg} \approx 10^{-30} \text{ kg} \\ e &= 1.6 \times 10^{-19} \text{ coulomb} \approx 10^{-19} \text{ coulomb} \\ \epsilon_0 &= 8.85 \times 10^{-12} \text{ coulomb}^2/\text{ntm}^2 \approx 10^{-11} \text{ coulomb}^2/\text{ntm}^2 \end{aligned}$$

For r , we may take the distance to be one Bohr radius,
 Viz. $r = a_0 = 5.29 \times 10^{-11} \text{ m}$

Thus we have,

$$\begin{aligned} V^2 &= (1/m_e) \times (1/4 \pi \epsilon_0) \times e^2/r \\ &= 1/(10^{-30}) \times 1/(10 \times 10^{-11}) \times (10^{-19})^2/(5 \times 10^{-11}) \\ &= 1/5 \times 10^{-38}/(10^{-51}) \\ &= 0.2 \times 10^{13} \\ &= 2 \times 10^{12} \text{ m}^2/\text{s}^2 \end{aligned}$$

Hence $V \approx 1.5 \times 10^6 \text{ m/s}$ Which is about 1% of the speed of light. Hence electron in an atom is really non-relativistic.



Problem 15

What is the total number of copies of newspapers printed in India everyday?

Hint:

How much is the literacy in our country ? What is the average weight of a popular daily newspaper?

Solution:

The population of our country is about 1000 million and the literacy about 60%. That is about 600 million people can read and write in our country. Generally, a newspaper is bought in a family. If a family consists of four on the average, there would be about 150 million "literate" families. We find generally that a large number of families, in fact about 60-70% are middle class families and remaining use public libraries etc. It would be safe to consider about 30% of 150 million or about 45 million copies of newspapers being sold everyday. This gives an average of about 1 newspaper per 15 literate people or 1 newspaper per 25 people in our country.

**Problem 16**

How many trees need to be cut off everyday to publish newspapers in our country?

Hint:

You need to know the number of copies of newspapers printed in our country (see problem No. 15). Now estimate the weight of a newspaper and then that of a full grown tree.

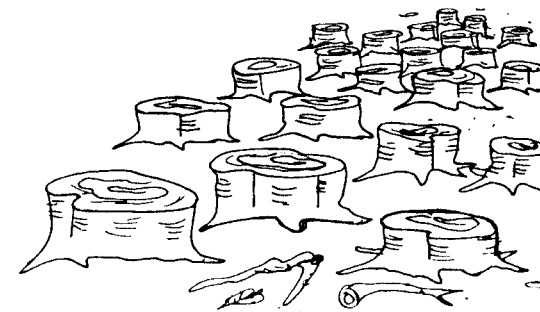
Solution :

From Fermi Problem No. 15, we have an estimate of 45 million copies of newspapers printed in our country daily. An average newspaper such as Times of India, Indian Express or the Statesman weights about 100 grams, i.e. 0.1 kg. Hence newsprint required to print 45 million copies would be about 4.5 million kg or 4500 metric tons.

An average sized tree can be considered about 5 metres high and diameter 0.3 metres. Its volume would be therefore.

$$\begin{aligned}\pi r^2 h &\cong 3 \times (0.3)^2 \times 5 \\ &\cong 3 \times 0.1 \times 5 \\ &\cong 1.5 \text{m}^3\end{aligned}$$

Taking specific gravity of wood to be 0.5 its density would be $0.5 \times 10^3 \text{ kg/m}^3$ (you may recall density of water is 10^3 kg/m^3). Thus weight of a tree with volume 1.5m^3 would be $1.5 \times 0.5 \times 10^3 \cong 1 \times 10^3 \text{ kg}$ or about 1 metric ton.



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Since 1 metric ton of wood gives only about 0.2 metric ton of newsprint, and the requirement of newsprint is 4,500 metric tons. As indicated above one tree weighs one metric ton and thus 90000 well grown trees need to be cut off everyday to print 45 million copies of newspaper.

23 Fermi Problems of The Art of Estimation

Problem 17

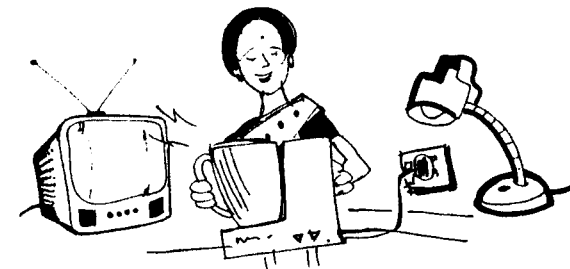
What is the consumption of electricity per day for domestic purposes in an Indian city with population three million?

Hint :

Recall what electrical appliances you use and their wattages. Consider how much time you keep them on. Then consider how many such connections should be there in the city.

Solution:

Let us consider a family to consist of four members. With a population of 3 million, this means there are about 8 lakh families in the city. Further, consider that each family dwells in a house with about 3 rooms and use electricity for about 10 hours with appliances of 300 watts (i.e. 2/3 lamps or tube lights about 40 watts, one fan about 100 watts, or a cooler or a refrigerator). Hence consumption of such a family would be 0.3×10 i.e. 3 kWh or 3 units per day. With 8 lakh families the consumption would be about 24 lakh units (kWh) per day



Problem 18

How much biogas energy could be produced every day through the human organic and other wastes in an Indian city with population 3 million ?

Hint :

Estimate the per capita waste produced from example of your house. From any popular book on energy find out how much organic waste is required to produce 1 m³ of biogas.

Solution:

Consider that per capita biological waste is about 1 kg per day. This would include the body wastes, vegetable wastes etc. Population being 3 million, human waste produced alone would be about 3 million kg per day. Now, 30 kg of organic waste produces about 1 m³ of biogas or methane. This implies that 3 million kg of human waste can produce about 1 lakh m³ of methane. Now 1 m³ of biogas can meet the fuel requirement of a family of 3-4 members for one day. Hence biogas produced through human organic and other wastes per day can meet daily fuel requirement of 4 lakh people (1 lakh families) in population of 3 million (8 lakh families).

**Problem 19**

How long a line can be drawn using a new ball point refill?

Hint :

In your normal writing, try to consider the length of each character you write and approximate speed and time for which you can write.

Solution:

In normal hand-writing each character can be considered to be about 1 cm in length. In addition consider that you can write about 10 foolscap pages a day and that there are 30 lines in one page with about 5 words per line of 5 letters each. Thus every day you draw a line of 75 metres length. At this rate (which is my speed), a jotter refill lasts for about 3 months or, say 100 days. Hence you can draw a line 7500 metres, i.e. 7.5 km long with a jotter refill!



Problem 20

How much thickness of motor-cycle/scooter tyre is lost during each turn?

Hint :

What are the dimensions and thickness of a scooter tyre? How many kilometres can it last?

Solution:

Consider the diameter of a scooter tyre to be about 30 cm. Hence its circumference would be approximately 1m. When a tyre wears out, let us take its tread to lose a thickness of 5 mm (5×10^{-3} m) in 50,000 km.

With this data, it can be seen that during the life time of a tyre the thickness lost per kilometre is $5 \times 10^{-3}/50,000 = 10^{-7}$ m/km. The circumference of the tyre being 1m, it turns 1000 times in 1 km. Hence amount of thickness lost every time the scooter wheel turns once is $10^{-7}/10^3 = 10^{-10}$ m.

If one recalls basic atomic data, the thickness lost per turn of the tyre is really of the order of a molecular diameter (Bohr radius a_0 of hydrogen atom is 0.53×10^{-10} m). A molecular layer is lost everytime a scooter/motor cycle wheel turns once from you tyre!

**Problem 21**

Given a very large sheet of paper 0.1 mm thick. Suppose one folds it 25 times. What will be the thickness of the pile?

Hint :

What is the thickness of the paper if folded once? Twice?

Solution:

Consider the thickness of the sheet of paper to be 0.1 mm, i.e. 10^{-4} m. Each time we fold, the thickness becomes twice. That is, if folded once, the thickness becomes 2×10^{-4} m, if folded twice the thickness becomes $(2)^2 \times 10^{-4}$ m and so on. If folded 25 times, the thickness would be $(2)^{25} \times 10^{-4}$ m.

Now $2^{10} \cong 10^3$ and $2^5 \cong 30$. Hence the thickness will be

$30 \times 10^6 \times 10^{-4} = 3000$ m or 3 km approximately!

In reality, how many times can you fold a sheet of paper? Try!

Problem 22*

Estimating the circumference of the Earth—another way.

Hint :

Do you know how 'metre' was defined?

Solution :

The metre was so defined by the French Academy (Napolean's time) so that it is 10^{-7} of quadrant of the Earth.

So the circumference of the Earth is 4×10^7 metres.

40,000 km.

Problem 22 a

What is the distance on the Earth's surface corresponding to 1° of latitude?

$10^7 / 90 = 111$ km per degree of latitude.

* Contributed by Prof. P.R. Pisharoty, Physical Research Laboratory, Ahmedabad.

Problem 23*

Determining the number of molecules in the atmosphere—ANOTHER WAY.

Hint:

Knowing the air pressure, can you estimate the mass of atmosphere? Do you know the gram molecular weight of air?

Solution:

1gm mole (28 gm of air) has 6×10^{23} molecules

Atmospheric pressure is 1 kg wt per cm^2

(76 cm of Hg \times 13.6 gm/cc)

So mass of atmosphere would be

Area in $\text{cm}^2 \times 1000$ gm.

No of molecules in the atmosphere

$$= \frac{\text{Area} \times 1000}{28} \times 6 \times 10^{23}$$

Surface area of the Earth can be derived as follows,

$$\begin{aligned} \text{Area} &= 4\pi r^2 = \frac{(2\pi r)^2}{\pi} = \frac{(4 \times 10^7 \times 10^2)^2}{\pi} \\ &= \frac{16}{\pi} \times 10^{18} \cong 4 \times 10^{18} \text{ cm}^2 \end{aligned}$$

(You may realize that $2\pi r$ is the circumference of the Earth, which is 4×10^7 m., see problem 22).

Hence number of molecules would be

$$= \frac{4 \times 10^{18} \times 10^3}{28} \times 6 \times 10^{23} = 10^{44}$$

* Contributed by Prof. P.R. Pisharoty, Physical Research Laboratory, Ahmedabad.

problems. Do not try to look up hints and solutions before giving an honest try, lest you feel deprived of the sense of pleasure and pride that accompanies creativity!

Problem 24

Estimate the least number of molecules in a gram of musk.

Hint :

The fragrance of musk can be detected at a distance of 500 meters from source in still air. A gram of musk sublimes in about a month.

Assume the area of a human nostril to be a square cm. And that at least one molecule should enter the nose per second to create the sensation of smell

Solution :

You can readily infer that

$$\frac{1}{24 \times 60 \times 60 \times 30} = \text{gms of musk sublimes per second}$$

If there are N molecules in a gram of musk, then N divided by $4\pi (500 \times 100)^2$ should be 1 molecule at least!

$$\text{i.e. } \frac{N}{86400 \times 30} \times \frac{1}{4\pi \times (500 \times 100)^2} = 1$$

$$\begin{aligned} \text{Hence } N &\cong 10^5 \times 10^2 \times \pi \times 25 \times 10^8 \\ &\cong \pi \times 25 \times 10^{15} \\ &\cong 10^{17} \end{aligned}$$

(This was done by a British scientist before Avogadro's number was computed)

**Contributed by Prof. P.R. Pisharoty, Physical Research Laboratory, Ahmedabad.*

Some more Fermi Problems for you!

1. What is the mass of the Earth?
2. Estimate the number of TV sets in you city.
3. Estimate the number of two wheelers in you city.
4. Determine the mass of the Sun.
5. To prevent loss of water due to evaporation from a lake measuring 50 km x 20 km, some quantity of oil is spread over its surface that forms a film of about one molecular diameter. How much quantity of the oil will be required for this purpose?
6. Determine the number of grass blades in a lawn with dimensions 200m x 100m.
7. Determine the molecular speed in air.
8. How much electrical power is required to move a train initially at rest?
9. What is the period of revolution of a satellite of Earth at a height of 1000km?
10. A high frequency radio wave makes a round trip of Earth by multiple reflections from ionosphere and ground. How much time does it take?
11. At home, your news paper vendor knocks at your door to collect the payment for one month's bill. You do not have the requisite amount, but, you discover that there is a jar (such as Bournvita or Horlicks) containing 50 paise coins. Can you quickly estimate whether the payment could be made?
12. Have you visited Mumbai anytime? If yes, how many trips are made by the suburban trains in a day there? Also estimate the number of suburban trains in Mumbai the railways have.

13. How long should a mission to Pluto take in a space- ship from Earth?
14. How much food does a man eat in his life time?
15. To one of the legendary questions by Akbar the Great, demanding of Birbal the number of crows in his kingdom, Birbal is said to have instantly answered off-hand a huge figure such as 10 million or so. Suppose the same question is posed to you, what figure would you quote? You may consider your own state.
16. Find the number of hairs on you head ! (if there is none, you may try your friend's head).
17. Determine the size of the solar array for a geosynchronous satellite such as INSAT-1B that generates about 1 KW of power.
18. How many people go abroad everyday from our country?
19. Determine the percent average growth of population every year in India.
20. Determine how much coal a thermal power station uses per day generating about 200 Megawatts of power.
21. How many graduates, people holding first university degree, do we produce in our country every year?
22. Find the length of thread in one normal sized reel.
23. During a war, some ten million refugees entered out country from a neighbouring one. How much land will be required to rehabilitate them?
24. What is the weight of air in the drawing room of your house?
25. What is the number of illiterate women in our country.